**CFA 7 Derivatives**

**7.1 Derivative instrument and market features**

Derivative: Security which derives value from another variable or security (the underlying)

Forward contract: Specify price to buy/sell a security at a future date

* Forward price: The price
* Settlement date: Date of future transaction
* Neither party pays at the initiation of the contract

The buyer of the forward gains if market price > forward price

* Has long exposure to the underlying

The seller of the forward gains if market price < forward price

* Has short exposure to the underlying

Deliverable contract: Payment and shares must be exchanged at the settlement date

Cash-settled contract: Only gains and losses are exchanged at settlement date

Hedging: Existing risk is transferred to another party

Speculating: Increasing risk

Benefits over simply selling the shares (cash market transaction)

* Gain exposure to a risk at low cost
* Transaction costs can be lower
* Initiating a derivative contract will have less of an impact on the market price of the underlying

Underlying can be bonds, indexes, currency, interest rate, commodities, credit default swaps, weather

Golden rule of futures: Do in the futures market what you must do in the future

* E.g., Sell futures if you want to sell the asset in the future

**Exchange traded Derivative markets**

Exchange traded derivatives

* Standardised
* Backed by central clearinghouse
* Rules set by exchange

Benefits of standardisation

* Improves liquidity, transparency, and transaction costs
* Facilitates clearing and settlement

Central clearinghouse (CCH)

* Does novation (takes the opposite position of each trade), guaranteeing the payments are made (reduces counterparty risk)
* Requires deposits from both participants when initiated, and more deposits for accounts the decline in value

Traded by exchange members

* Exchange members (dealers, market makers) make money from bid-ask spreads

**OTC markets for Derivatives**

OTC: Dealers with no central location

* Forwards, swaps, some options are traded OTC
* Unregulated, less transparent, higher transaction costs
* Can have no CCH

Contracts can be customised

Central counterparty (CCP): Acts as a CCH

* Central Clearing Mandate requires this
* Dealers record their swaps on a swap execution facility (SEF), and the CCP replaces the trade with 2 trades
* Downside: Counterparty risks are concentrated rather than distributed

**7.2 Forward commitment and contingent claims**

**Forwards**

Forward contract: Buyer commits to buy an asset at a specific price on a specific date in the future

**Futures**

Futures contract: Similar to forwards, but standardised and exchange traded

* Futures trade in a secondary market, have more regulation, more transparency
* Require daily cash settlement of gains/losses

Margin is collateral both the buyer and seller must deposit (no loan, so no interest charge)

* Mark-to-market: Margin balance is adjusted for futures value

Initial margin: Amount of cash to be deposited for trade to be made

Maintenance margin: Min amount of margin required in a futures account

* Must be brought back to the initial margin if it dips below maintenance margin

Price limits: Exchange imposed limits on how much each day’s price can change from the previous day

* Banned to trade outside the range
* Circuit breakers: If the price reaches the limit, traded is temporarily suspended

**Swaps**

Swaps: Agreements to exchange a series of payments on multiple settlement dates over a specified period

* At each settlement, the payments are netted

Trade in dealer market – have counterparty risk if no CCP

Interest rate swap: Swap fixed interest rate payments for the floating MMR rate payments

* The fixed rate payer is essentially borrowing at a fixed rate and buying a floating rate bond
* Can hedge interest changes

Can be constructed with a series of forward contracts

**Credit default swap**

CDS: Protection buyer makes fixed payments on the settlement dates and protection seller only pays if the underlying has a credit event

* E.g., Default, bankruptcy

Seller must pay an amount to offset the loss in value

Fixed payments represents yield premium for expected loss on defaults

**Options**

Put: Right to sell the underlying for a specific price for a specified period of time

* Put buyer has right to sell
* Put seller must buy if exercised

Call: Right to buy the underlying for a specific price for a specified period of time

* Call buyer has right to buy
* Call seller must sell if exercised

Strike price (aka exercise price): Specified price

Time to expiration: Specified period of time

Option premium: Price of an option

is the strike price

is the underlying price at expiration

is the option premium

Value of a call option to buyer is

Value of a put option to buyer is

Value to the seller is essentially the negative for the buyer

Profit for buyer is the option payoff minus the option premium

Profit for seller is the option premium minus option payoff

Buyer max loss for calls and puts is the option premium

Seller max loss for calls is unlimited, max loss for puts is

Buyer max gain for calls is unlimited, max gain for puts is

Seller max gain for calls and puts is the option premium

Always a zero sum gain

If someone “writes” an option, they are the seller

**Forward commitments vs contingent claims**

Forward commitment: Legally binding action for the future

* Forwards, futures, most swaps

Contingent claim: Claim that depends on an event

* Options, credit default swaps

**7.3 Derivative benefits, risks, and uses**

**Advantages**

Change risk exposure

* Can create risk exposures not available in cash markets

Provide information that cash market transactions do not

* Can estimate future price volatility expected of underlying
* Can be used to estimate the expected price of the underlying (or expected future interest rates)

Operational advantages

* Ease of short selling – easier to sell a future than to short underlying
* Lower transaction costs
* Greater leverage – cash required to take a position is much less
* Greater liquidity – from low cash requirements

Improved market efficiency

**Risks**

Implicit leverage

* Lower cash requirement increases leverage

Basis risk

* Underlying of derivative differs from position hedged by the derivative
* Investor horizon may differ from the settlement date

Liquidity risk

* Cash flows from hedge do not match cash flows of investor positions
* Need to meet all the margin calls during the life of the contract

Counterparty credit risk

* Options: Seller faces no risk, but buyer does
* Forwards: Both face risk
* Futures: CCH, initial margin deposit, daily mark-to-market reduce the risk

Systemic risk

* Financial markets may be impacted by widespread speculation

**Use by issuers**

Hedge accounting: Recognise gains/losses in derivative hedges at the same time as corresponding changes in the values of assets/liabilities being hedged

Cash flow hedge: Hedging domestic currency in a foreign currency using forwards

* Includes converting a floating rate liability to a fixed rate liability

Fair value hedge: Offsets changes in the values of assets/liabilities

* Swap fixed interest for floating interest on fair value reporting debt to make less sensitive to changes in interest
* Sell forwards on inventory valued at fair value to reduce inventory fluctuations

Net investment hedge: Reduces volatility of the value of equity of a foreign subsidiary on balance sheet

* Currency futures can be used

**Use by investors**

Examples

* Gain exposure to the price of an asset with low initial finds required
* Swap fixed rate for floating rate
* Can modify portfolio risk by buying/selling index derivatives
  1. **Arbitrage, replication, and cost of carry in pricing derivatives**

Arbitrage: Buy one asset and sell an identical one at a higher price (risk free gain)

No-arbitrage condition determines derivative value

* If there is arbitrage it will be exploited very quickly

Transaction costs means small arbitrage can exist that is less than the transaction costs

is the current price

is the forward price at

is the coupon

Forward price is given by rearranging:

No arbitrage is given by:

Replication: Creating a portfolio of cash market transactions that has the same payoffs as a derivative

* Long forward: Borrow the current price of asset at the interest rate and purchase the asset
* Short forward: Short the asset and invest the proceeds at the interest rate

There can be costs and benefits to holding the asset

* Costs: Storage, insurance, spoilage (for commodities), low for financial assets
* Benefits: Convenience yield, shortage of an asset, dividend yield

is the present value of costs

is the present value of benefits

No arbitrage including costs and benefits:

With continuous compounding:

is rate of cost compounding

is rate of benefit compounding

**Forward contracts on currencies**

No arbitrage price on forward currency (in economics):

**7.5 Pricing and valuation of forward contracts for an underlying with varying maturities**

**Value of a forward**

Value of a forward contract at initiation is 0

The value of a forward at time for buyer:

Essentially:

* Value for buyer: Spot minus PV of forward contract
* Value for seller: PV of forward contract minus Spot

At expiration, it is the same, except it is no longer PV

**Forward rates**

Forward rates: Yields for future periods

2y1y or is a 1 year loan, made 2 years from now

Implied forward rate is the forward rate where the 2 strategies have the same yield

1. Investing from to the forward date and rolling over the proceeds for the period of the forward by reinvesting
2. Investing from until the end of the forward period

Implied forward rate can be found using:

Forward rate agreement: Fixed rate payer (long) pays the forward rate, floating rate payer (short) pays a future reference rate on the same size of principal

* Used to manage volatility of interest sensitive assets/liabilities

**7.6 Pricing and valuation of futures contracts**

Price of a forward contract is constant over its life

* However, the value fluctuates with underlying 🡪 Payment at settlement reflects this

Price and value of futures contract both change daily

* When mark-to-market is settled

Interest rate futures are quoted on a price basis

Interest rate futures price for a MRR from time to :

Basis point value (BPV) of an interest rate future is given as:

0.01% is 0.0001

Period is given as annual

**Forward vs future prices**

Forwards have no mark-to-market 🡪 Gains/losses settled at expiration

Futures have mark-to-market 🡪 Gains/losses settled daily, requires margin

If interest rates are constant or uncorrelated with futures prices:

* 🡪 Future and forward prices are the same

If positive correlation between interest rates and futures prices:

* 🡪 Excess margin when rates are high which can be reinvested, needs margin deposits when rates are low and opportunity cost is less
* Futures more attractive than forwards when positively correlated, less attractive when negatively correlated

Differences in forwards and futures not observed in practise

* Due to short maturity of forwards, and availability of funds at risk free rates

Convexity bias: Value of interest rate forwards have convexity

* Increase in rates decreases the forward’s value by less than a decrease in rates increases a forward’s value
* Can lead to difference between forward and future prices

**7.7 Swap Valuation**

Example: Swapping a fixed rate for a floating rate

Can separate payments into 1 known payment, and several unknown payments

* Unknown payments are essentially FRAs (forward rate agreements)
* 1st payment is the known one

In each period, the fixed rate payer receives

Each individual FRA are not 0 value contracts at initiation

Par swap rate: Swap fixed rate that gives the swap a 0 value at initiation

* Makes the individual FRAs sum to be 0

**Price and value of swaps**

Price of a swap: Fixed rate of interest (the par swap rate) specified in the contract

* Doesn’t change

Par swap rate can be found from:

is the current effective spot rates

are the forward rates implied by spot rates

is the fixed rate payment

Value of fixed rate payer is PV of expected future floating payments minus PV of future fixed payments

* Increasing future expected rates increases value of fixed rate payer position
* Decreasing future expected rates decreases value of fixed rate payer position

**7.8 Pricing and valuation of options**

**Option definitions**

Moneyness: Can be In-the-money or Out-the-money

* In-the-money: Immediate exercise will have positive payoff
* Out-the-money: Immediate exercise will have negative payoff
* At-the-money: Immediate exercise has no gain/loss

Exercise value: Max of zero or the In the money option

Time value: Exists prior to expiration

Option premium is given by:

**Upper and lower bounds (need to know)**

Initial values of options are positive

Premium paid limits losses to buy, and limits gains to seller

is the price of a European call option

is the price of a European put option

is time left to expiration

European options cannot be exercised before the expiration date

Upper bound for call options:

* Max value is the stock price

Upper bound for put options:

* Since European puts cannot be exercised before expiration, their max value is the exercise price discounted at the risk free rate

Lower bound for call options:

Lower bound for put options:

Imagine you are paying for the option with a discounted bond at the risk free rate

**Factors that determine the value of an option**

6 factors

|  |  |  |
| --- | --- | --- |
| **Increase in:** | **Value of Call** | **Value of Put** |
| Price of underlying asset | Increase | Decrease |
| Exercise price | Decrease | Increase |
| Risk free rate (think of PV of the future cash flow) | Increase (less expensive to pay) | Decrease (less received from payment) |
| Volatility of underlying | Increase | Increase |
| Time to expiration (essentially increases volatility) | Increase | Increase (exc some European puts – if in the money, may get discounted too much) |
| Costs to holding underlying asset | Increase | Decrease |
| Benefits to holding underlying asset | Decrease | Increase |

**7.9 Option replication using put-call parity**

**Put call parity**

Assume European style options

Put call parity: Based of a fiduciary call and a protective put

Fiduciary call: Combination of a call with exercise price and a discount riskless bond that pays at maturity

* Payoff when in the money:
* Payoff when out the money:

Protective put: Share of stock with a put option on the stock

* Payoff when in the money:
* Payoff when out the money:

If , both pay

If both pay

The put-call parity relationship is (need to know):

This can be rearranged to give the synthetic payoffs (, , )

* E.g., Creating the stock price with the other components

**Put call forward parity**

Derived from forward contract rather than the underlying asset

Put-call-forward parity is given as:

Can be rearranged to:

**7.10 Valuing a Derivative using 1 period binomial model**

**Binomial model**

Binomial model: The value will change to one of two possible values in the next period

Need to have:

* Value of underlying at beginning, exercise price, risk free rate
* Returns for an upward move, or downward move
* No probabilities required

Hedged portfolio has the same value for an upward move and a downward move

* Portfolio’s initial value grows at the risk free rate

Hedge ratio: No. of underlying shares to buy for each call option to achieve hedged portfolio

To value a call option: Create a portfolio that is short a call and long the stock

To value a put option: Create a portfolio that is long a put and long the stock

Calculation: Set the value of the portfolio from upwards move equal to value of the portfolio from downwards move

* Value of portfolio is given by no. of shares (unknown) multiplied by price, minus call value or plus put value
* Rearrange for no. of shares (hedge ratio)
* Then find value of portfolio (future one) and discount it by discount rate to find current portfolio value
* Then can find value of put or call option

**Risk neutrality**

is the upward move factor

is the downward move factor (the reciprocal of )

is the risk neutral probability of an upward move

is the risk neutral probability of a downward move:

With the probabilities find the EV of payoffs (remember the min is 0 for either one as they are options)

* Then discount it with the risk free rate for the PV of the option

Can use risk free rate as it is based on no arbitrage